Chapter 27: Electromagnetic Induction
Thursday October 27th

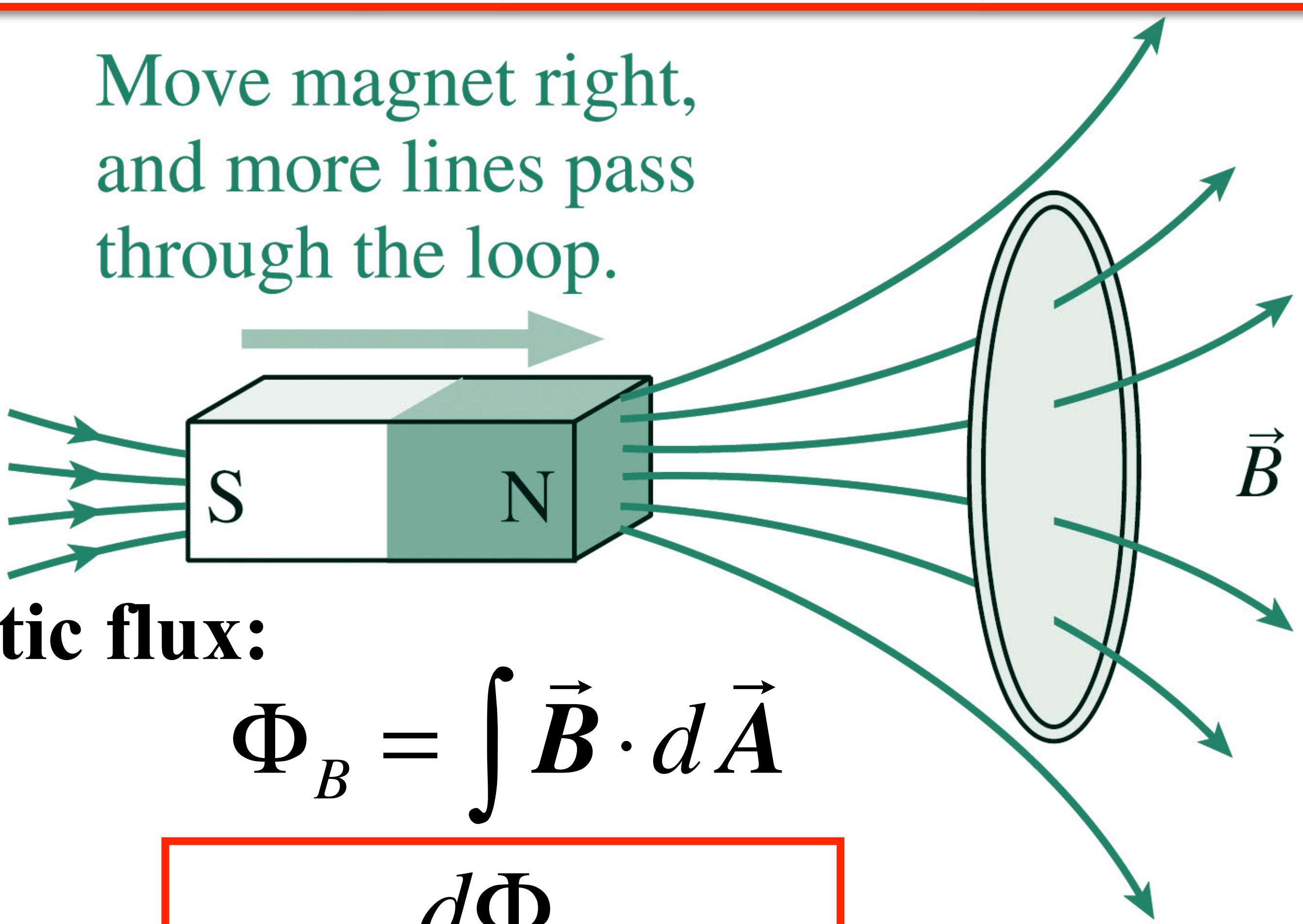
- **Normal lab schedule this week**
- **Mini-exam 4 next Thursday**
  - **Will cover end Ch. 26 and Ch. 27**

- Review of Lenz' law
  - Some cool demos
- Inductors
  - Analogy with capacitors
  - Back emf in inductors
  - Inductance of a solenoid
- *RL* circuits
  - Time dependent current (analogy with *RC* circuits)
  - Energy stored in inductors and magnetic fields

Reading: up to page 486 in the text book (Ch. 27)

Review: Faraday's Law

The induced emf in a circuit is proportional to the rate of change of magnetic flux through any surface bound by the circuit.



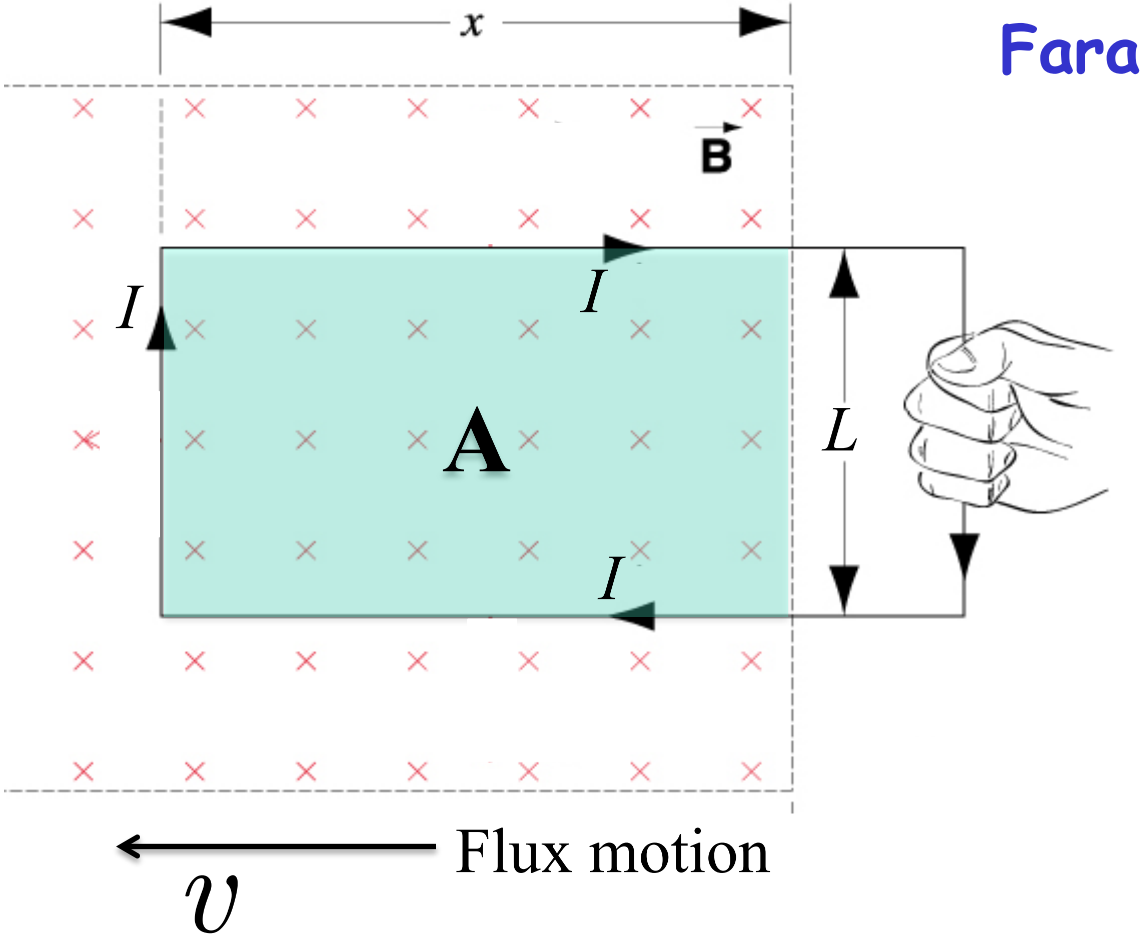
Magnetic flux:

$$\Phi_B = \int \vec{B}\cdot d\vec{A}$$

$$\boxed{\varepsilon = -\frac{d\Phi_B}{dt} = IR}$$

The minus sign turns out to be very important

Motional emf and the Lorentz Force Law



Faraday's law:

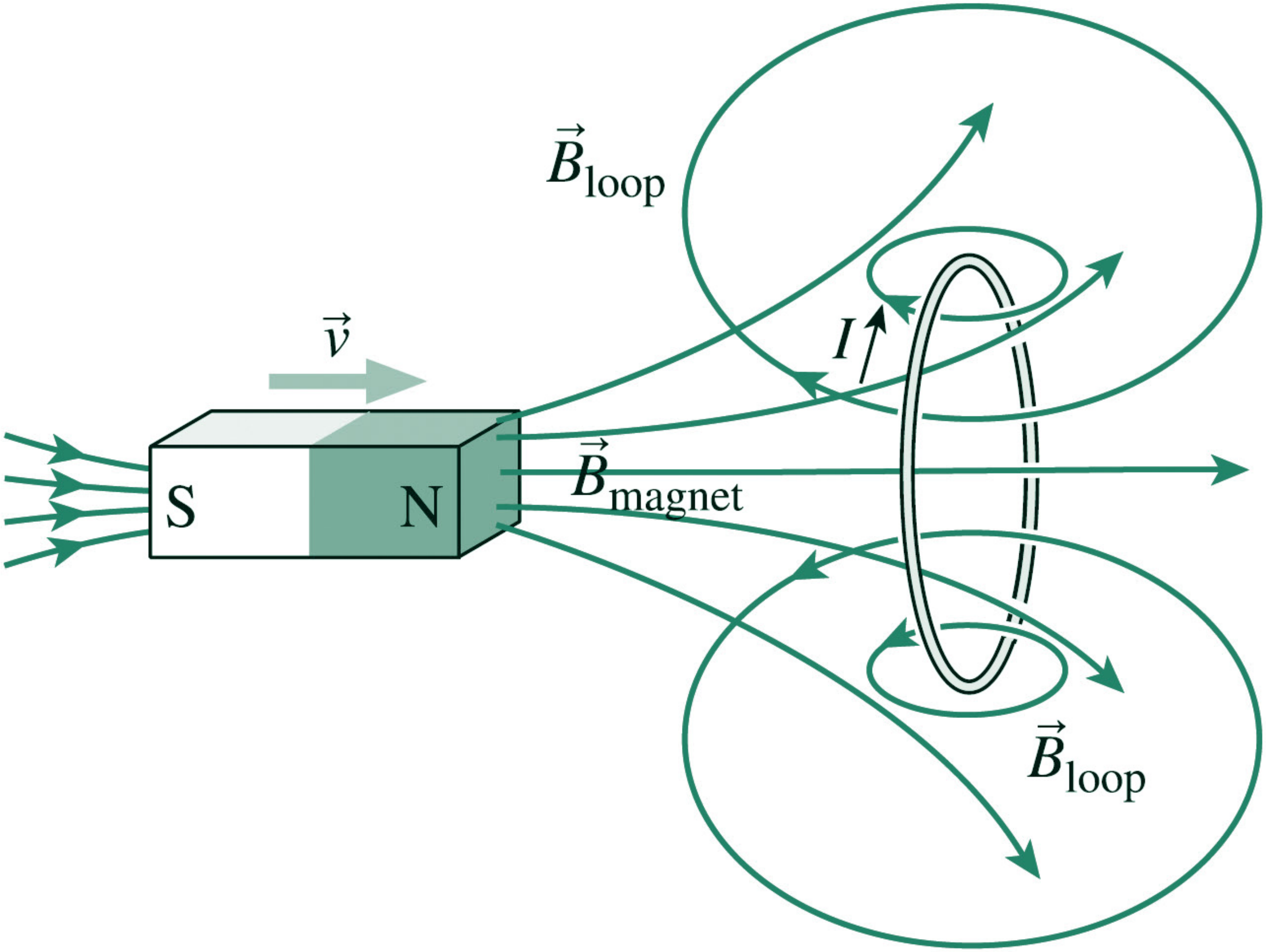
$$\Phi_B = BA = BLx$$

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = BLv$$

$$I = \frac{\varepsilon}{R} = \frac{BLv}{R}$$

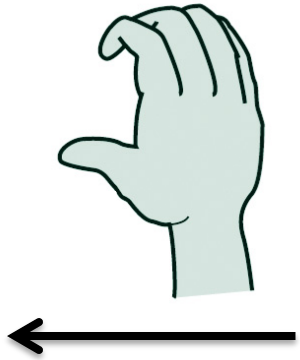
An example of relativistic invariance

Review: Lenz' laws



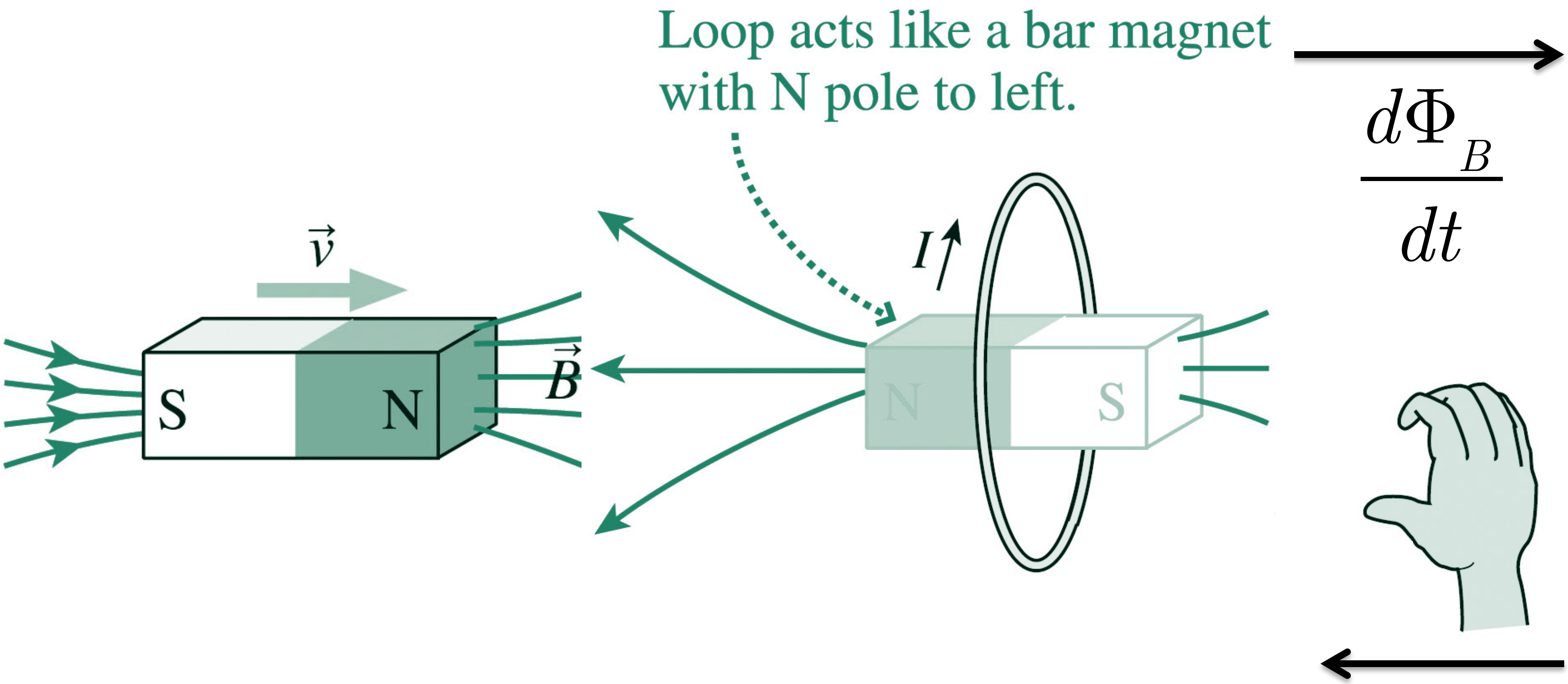
$$\xrightarrow{\hspace{3cm}}$$

$$\frac{d\Phi_B}{dt}$$



Right-hand rule with thumb this way

Review: Lenz' laws

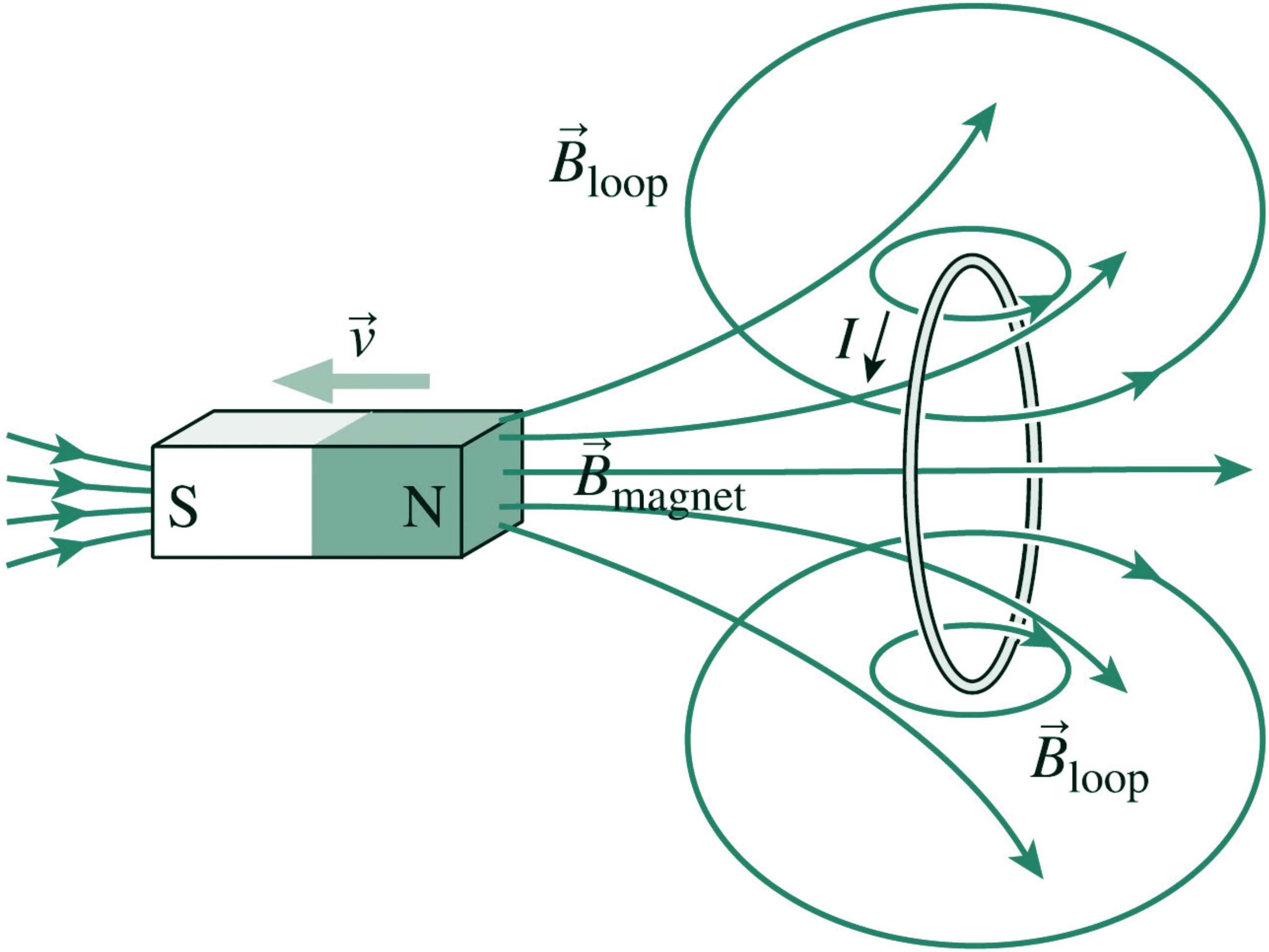

Loop acts like a bar magnet with N pole to left.

$$\frac{d\Phi_B}{dt}$$

Right-hand rule with thumb this way

Repulsive interaction

Review: Lenz' laws



←

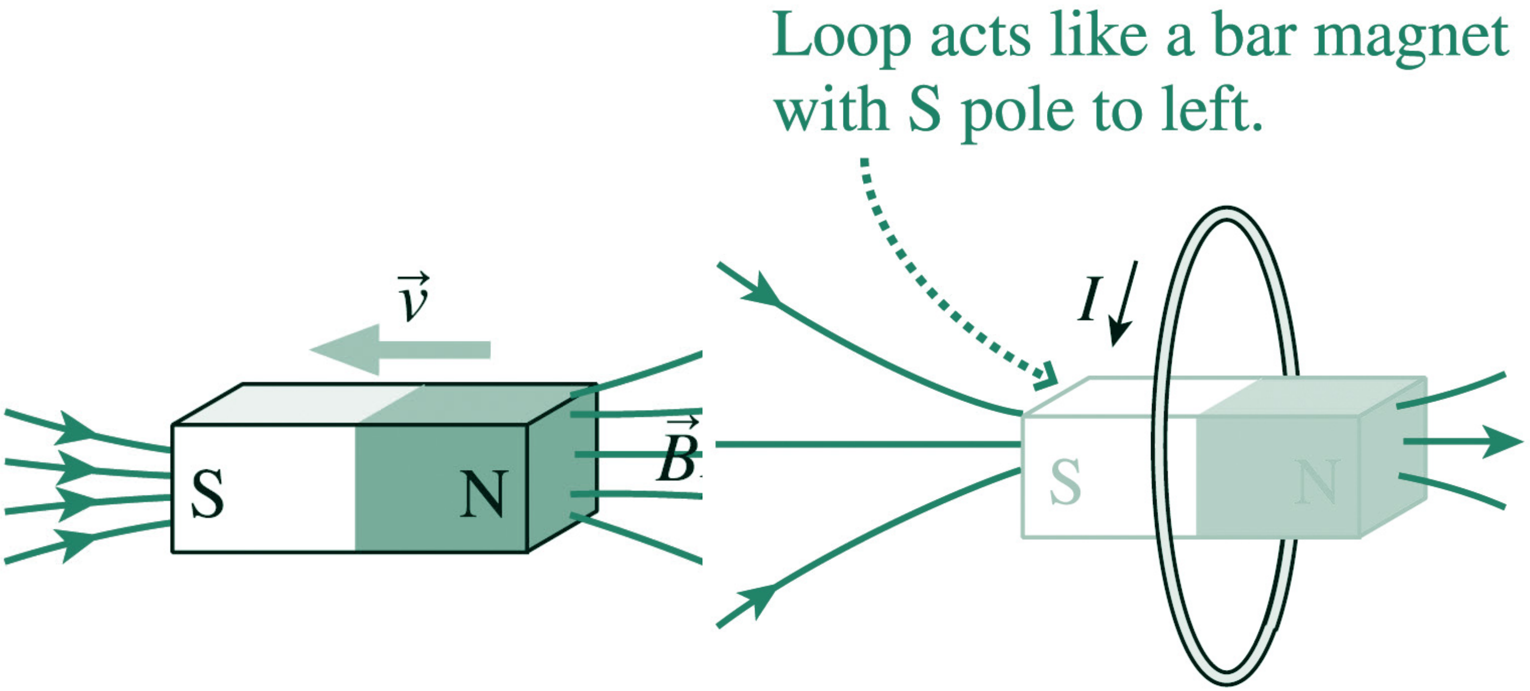
$$\frac{d\Phi_B}{dt}$$

→

Right-hand rule with thumb this way

Review: Lenz' laws

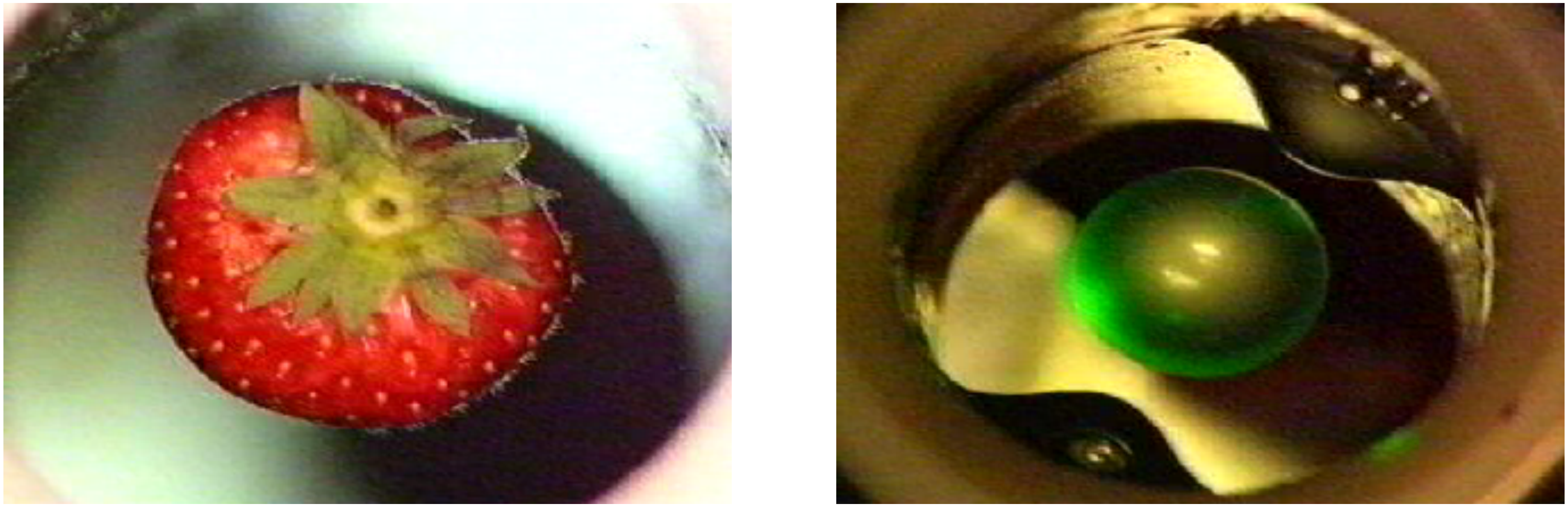
Currents will dissipate in a resistive material



$$\frac{d\Phi_B}{dt}$$

Right-hand rule with thumb this way

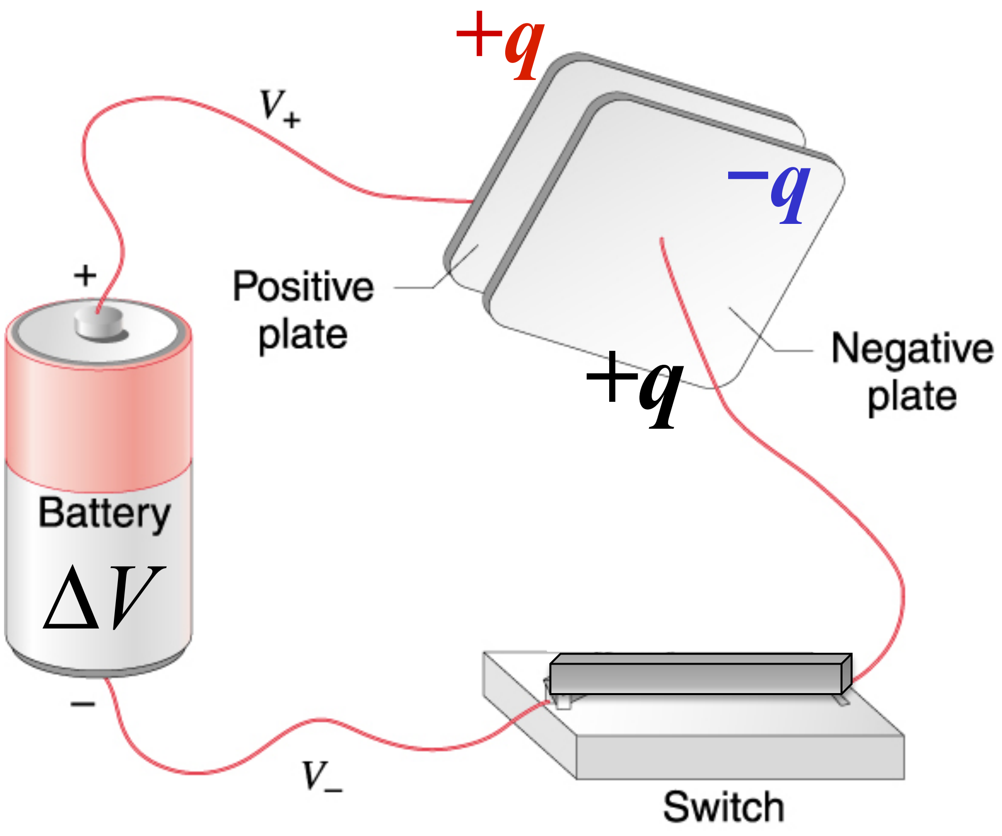
Attractive interaction
(always opposes change)

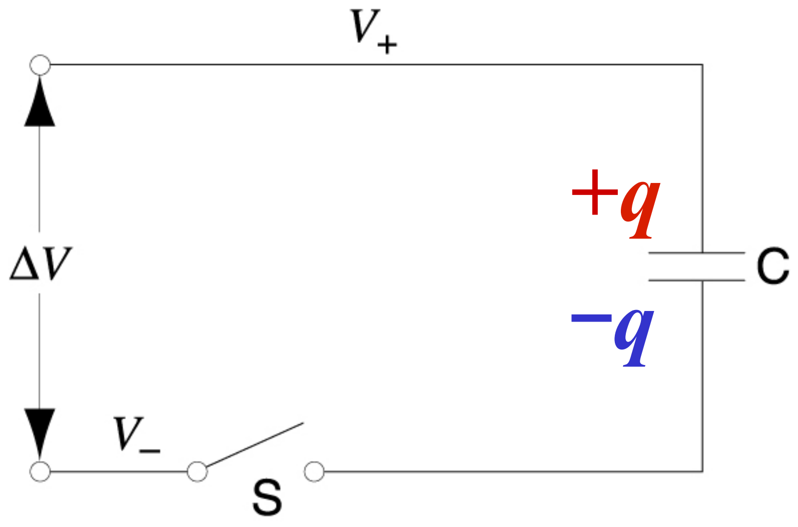


http://hypervocal.com/vids/2011/next-stop-hoverboards/

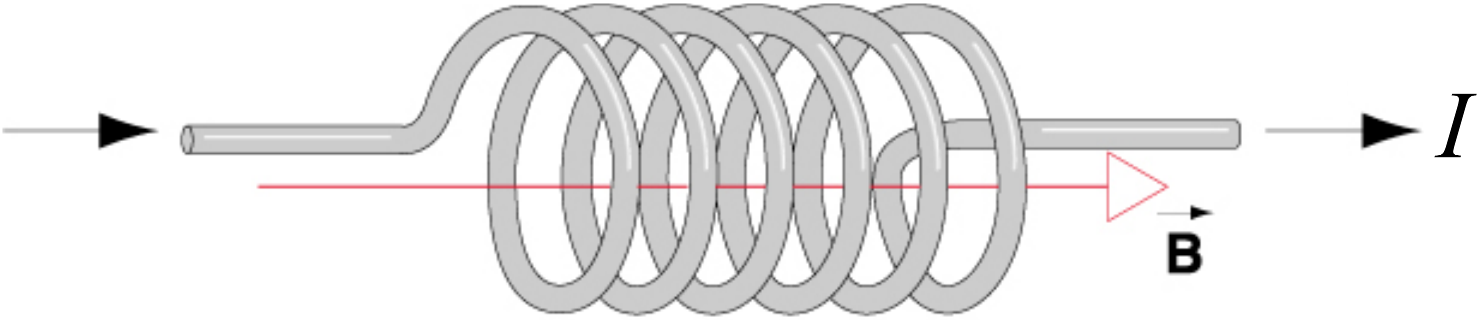
Inductors: the analogy with capacitors

- Used to store energy in electromagnetic fields [in contrast to batteries (chemical cells) that store chemical energy].
- Capacitors can release electromagnetic energy much, much faster than chemical cells. They are thus very useful for applications requiring very rapid responses.





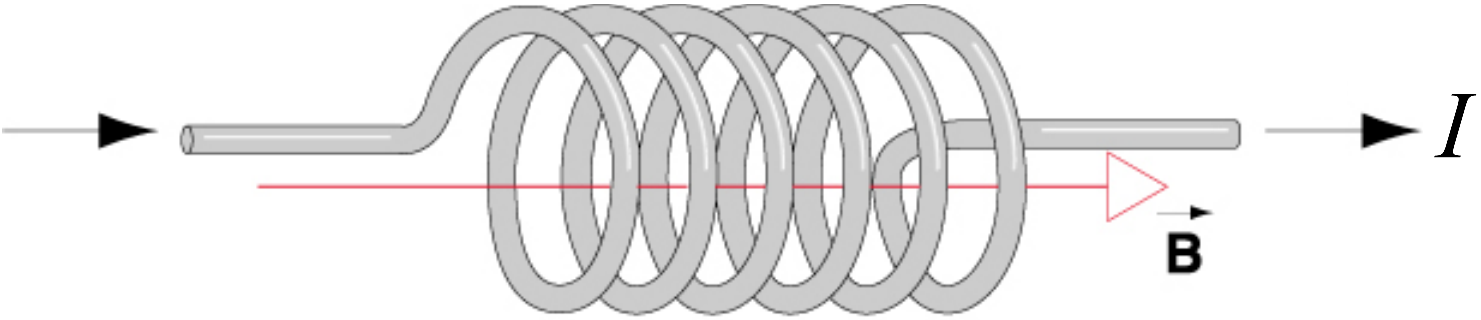
Inductors: the analogy with capacitors



- An increasing current creates magnetic flux.
- As this increasing magnetic flux threads the circuit, an emf is necessarily generated (Faraday's law), i.e.

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

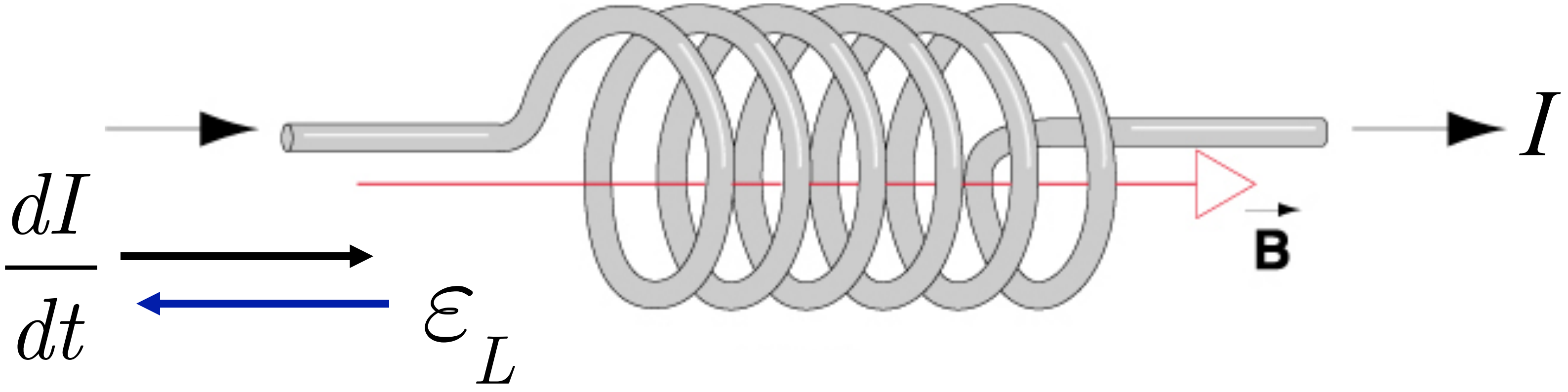
Inductors: the analogy with capacitors



- In circuits, we are more concerned with currents and voltages (including emf's) than flux.
- However, Φ_B is obviously proportional to the current I in the inductor. Thus, we can assume that

$$\varepsilon \propto \frac{dI}{dt}$$

Inductors: the analogy with capacitors



- We can, therefore, define a quantity L called inductance, which relates I to Φ_B and, thus, dI/dt and ε:

$$\boxed{\Phi_B = LI}$$

Units:
weber/amp
$T.m^2/A$
Henry (H)

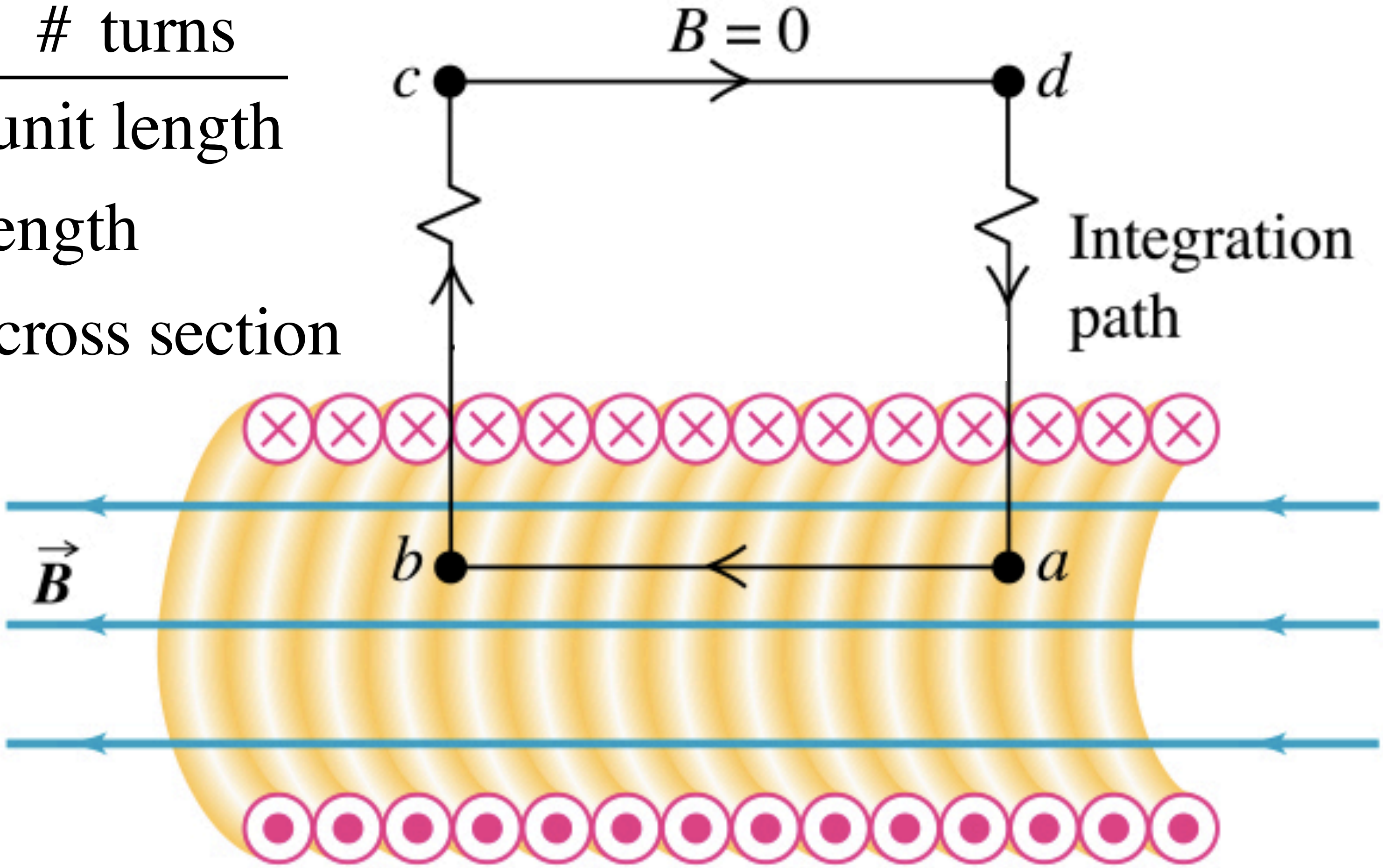
$$\boxed{\varepsilon_L = -L\frac{dI}{dt}}$$

Self Inductance of a Solenoid

$$n = \frac{\#\ \text{turns}}{\text{unit length}}$$

$l = $ length

$A = $ cross section



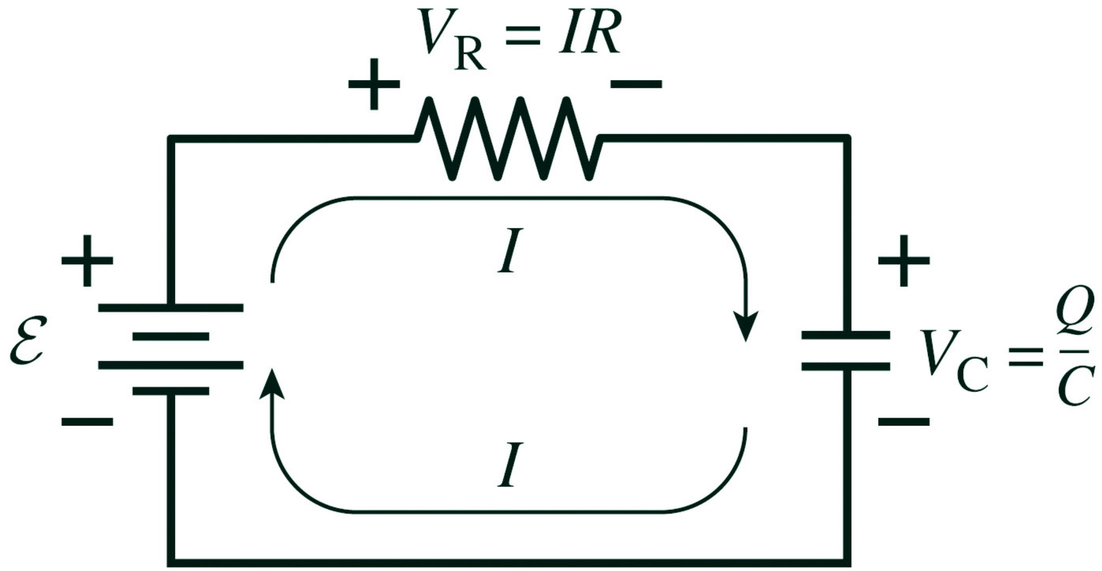
Central part of solenoid

Uniform field:

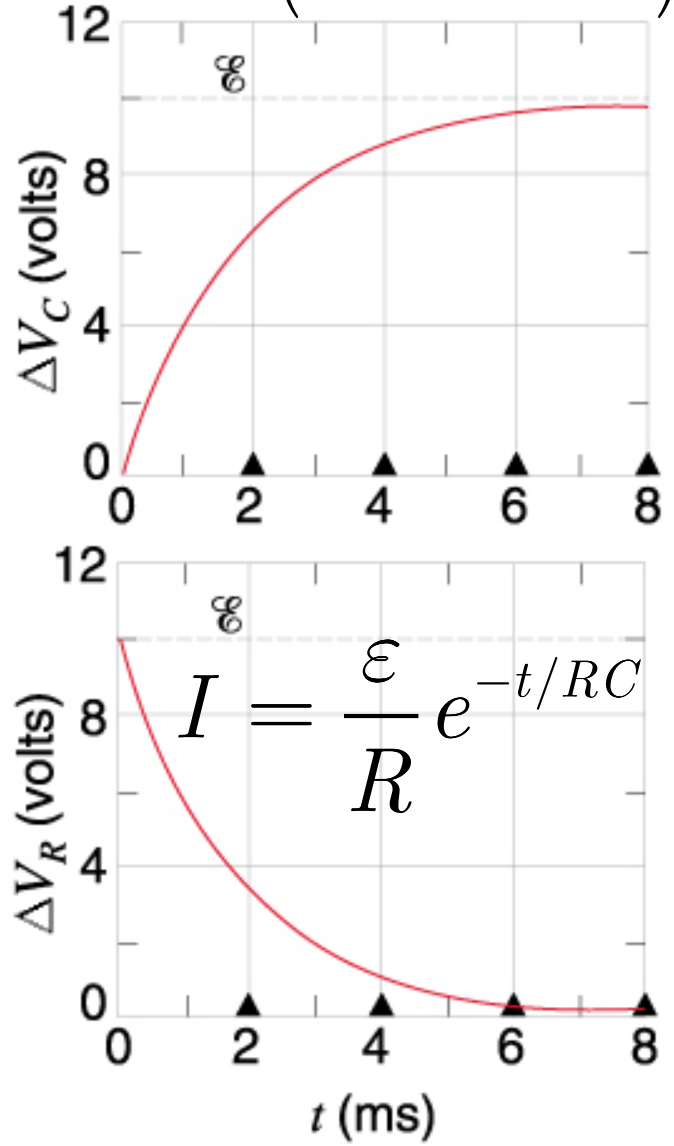
$$B = \mu_o nI$$

$$\boxed{L = \mu_o n^2 Al}$$

RC circuits (charging a capacitor)



$$Q(t) = C\varepsilon\left(1 - e^{-t/RC}\right)$$



$$I = \frac{\varepsilon}{R} e^{-t/RC}$$

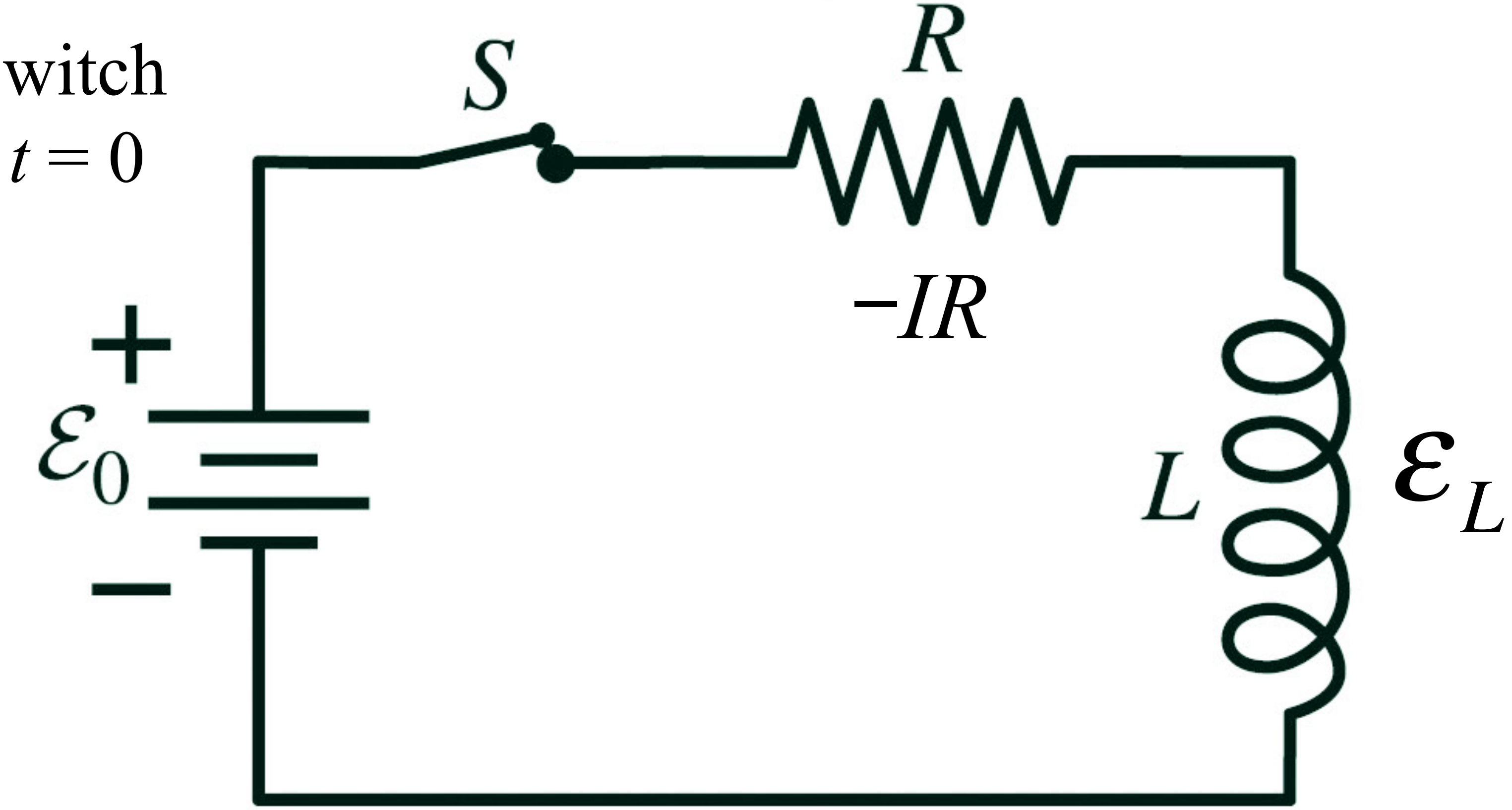
Kirchoff:

$$\varepsilon - IR - \frac{Q}{C} = 0$$

$$\varepsilon - R\frac{dQ}{dt} - \frac{Q}{C} = 0$$

LR circuits (similarity to *RC* circuit)

Close switch
at time $t = 0$



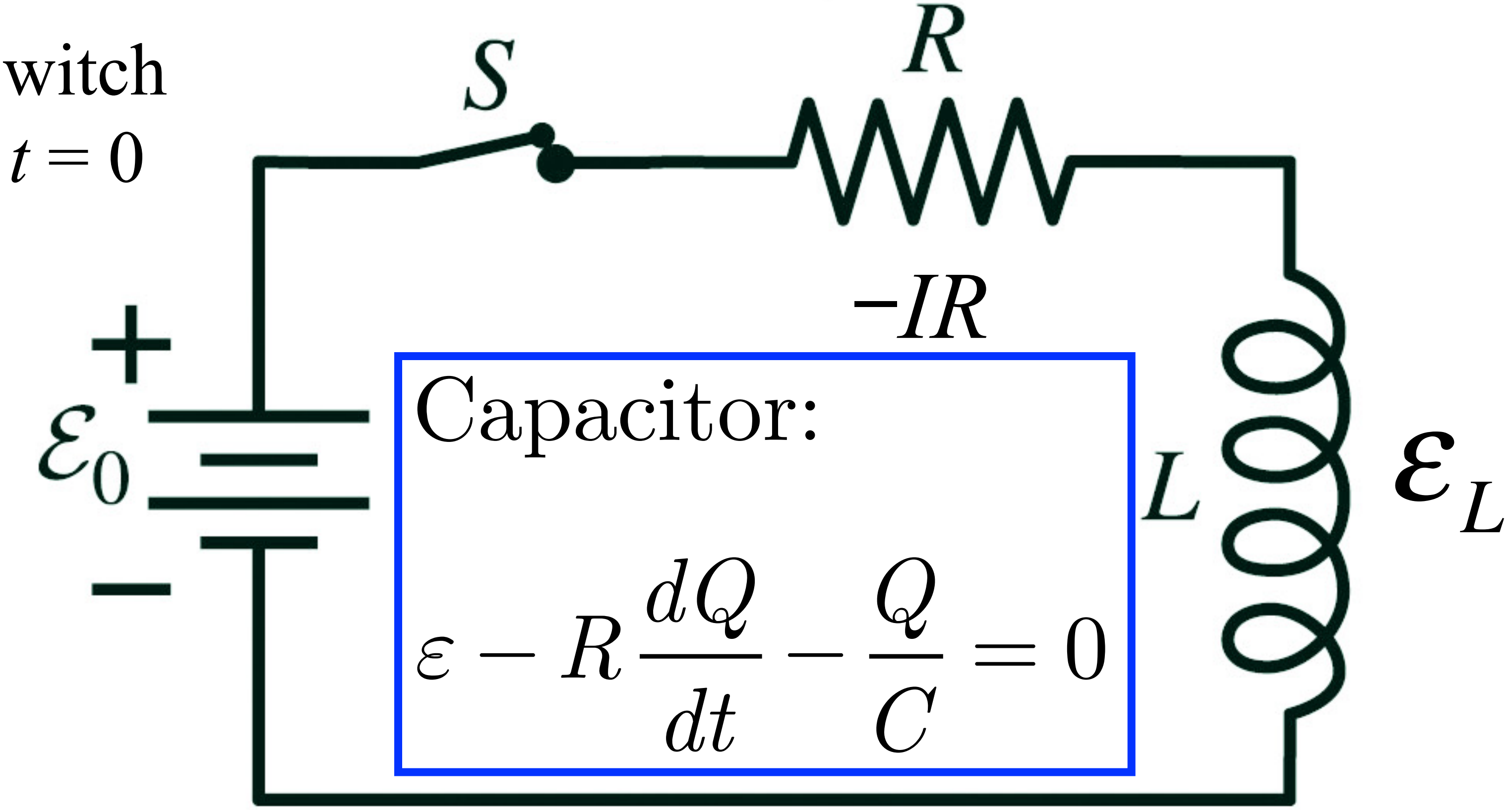
Kirchoff: $$\varepsilon_0 - IR + \varepsilon_L = 0$$

At $t = 0$, $I = 0$:

$$\varepsilon_0 + \varepsilon_L = 0 \quad \Rightarrow \quad \varepsilon_L = -\varepsilon_0$$

LR circuits (similarity to *RC* circuit)

Close switch at time $t = 0$



Capacitor:

$$\varepsilon - R\frac{dQ}{dt} - \frac{Q}{C} = 0$$

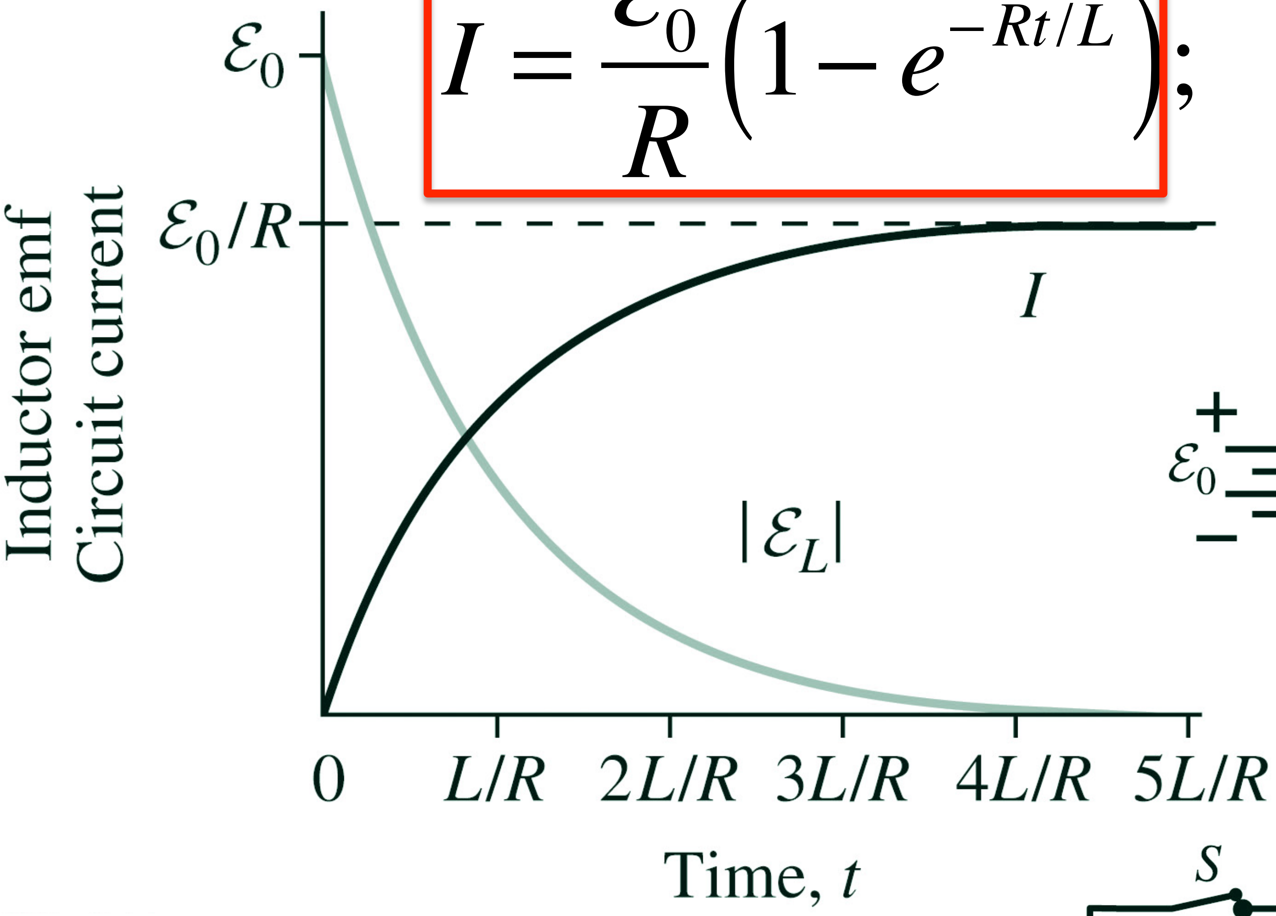
Kirchoff:

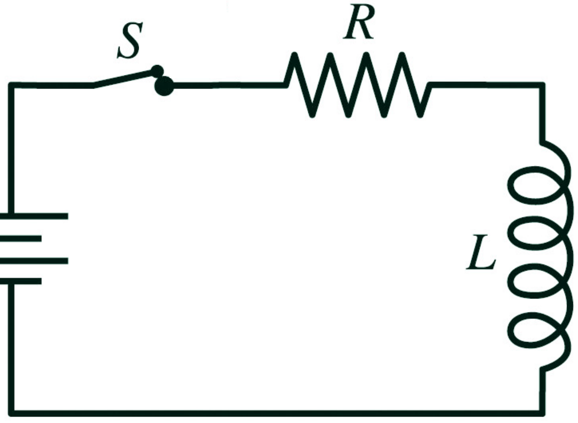
$$\varepsilon_0 - IR + \varepsilon_L = 0$$

$$\varepsilon_0 - L\frac{dI}{dt} - RI = 0$$

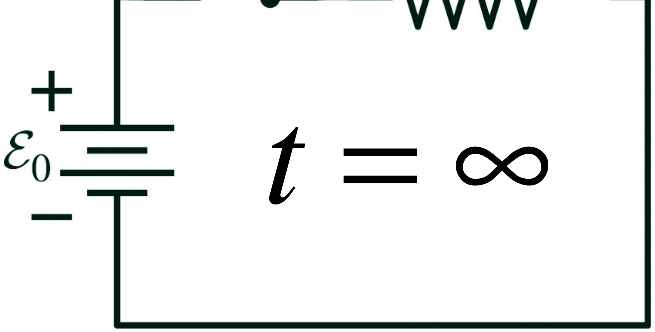
LR circuits (similarity to *RC* circuit)

$$\boxed{I=\frac{\mathcal{E}_0}{R}\left(1-e^{-Rt/L}\right)};\qquad \tau=\frac{L}{R}$$





$$\boxed{\varepsilon_L=-\varepsilon_0e^{-Rt/L}}$$

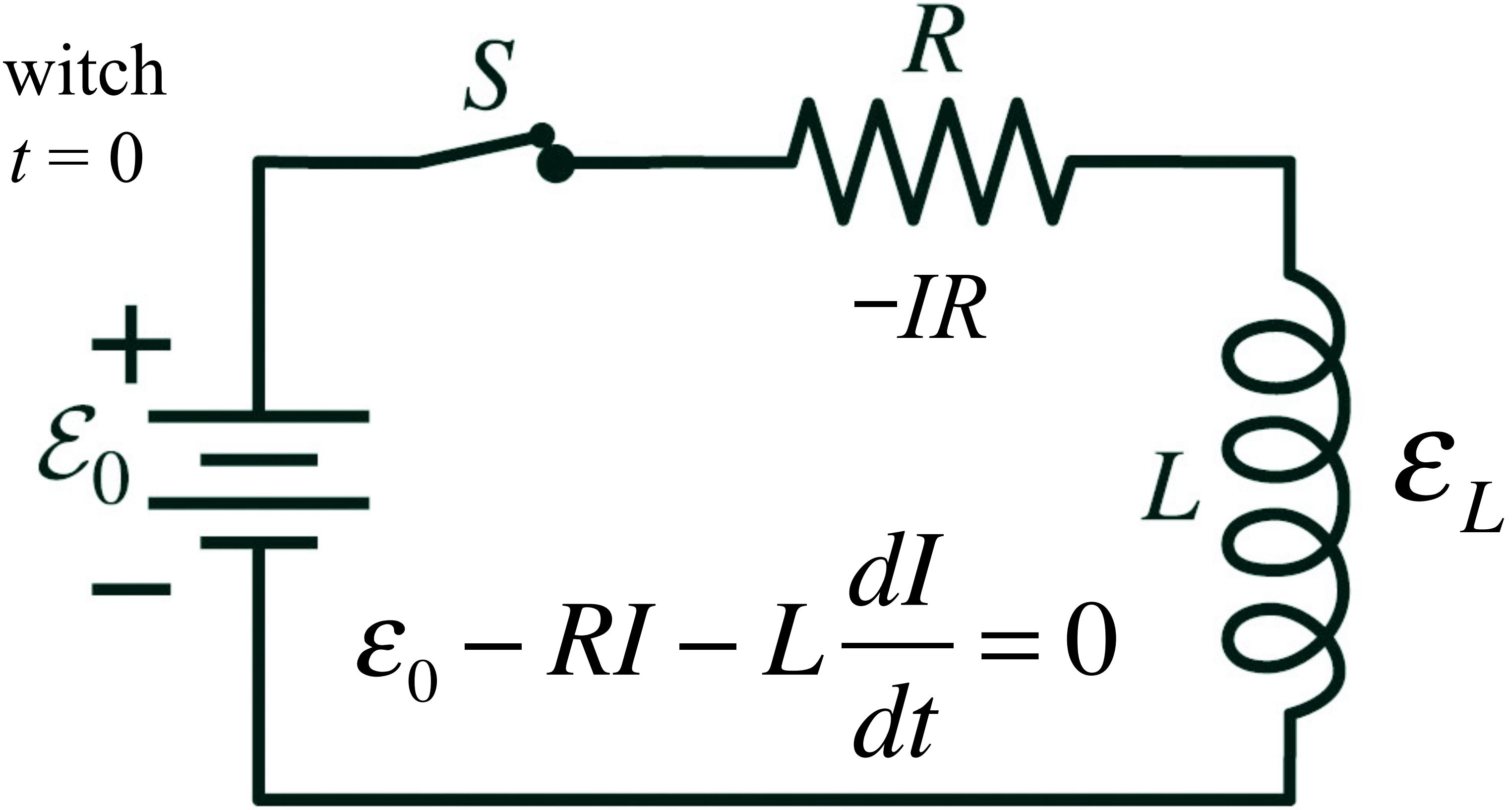


$$t=\infty$$

$$\varepsilon_L=0$$

Energy Stored in an Inductor

Close switch
at time $t = 0$



$$\varepsilon_0 - RI - L\frac{dI}{dt} = 0$$

Power:
$(\times\ I)$

$$\varepsilon_0 I - RI^2 - LI\frac{dI}{dt} = 0$$

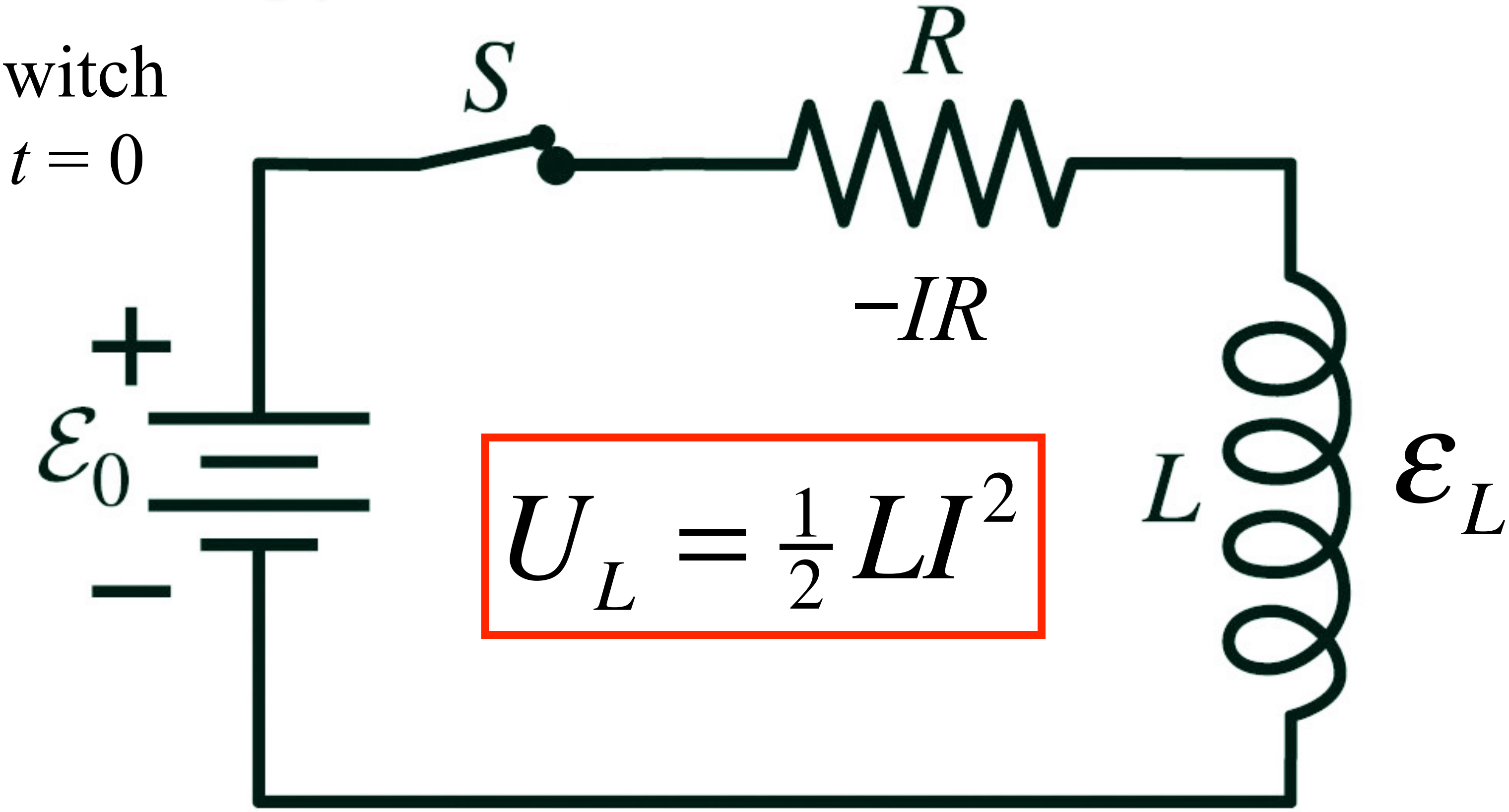
Supplied by battery

Lost in resistor

Stored in inductor

Energy Stored in an Inductor

Close switch
at time $t = 0$



$$\boxed{U_L = \tfrac{1}{2} L I^2}$$

Energy stored in magnetic fields:

Energy density: $$u_B = \frac{\text{Energy}}{\text{unit volume}} = \frac{B^2}{2\mu_o}$$